

Quantum Walks of Two Interacting Anyons in 1D Optical Lattices

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We investigate continuous-time quantum walks of two indistinguishable anyons in one-dimensional lattices with both on-site and nearest-neighbor interactions based on the fractional Jordan-Wigner transformation. It is shown that the two-body correlations in position space are symmetric about the initial sites of two quantum walkers in the Bose limit ($\chi = 0$) and Fermi limit ($\chi = 1$), while in momentum space this happens only in the Bose limit. An interesting asymmetry arises in the correlation for most cases with the statistical parameter χ varying in between. It turns out that the origin of this asymmetry comes from the fractional statistics that anyons obey. On the other hand, the two-body correlations of hard-core anyons in position space show uniform behaviors from anti-bunching to co-walking regardless of the statistical parameter. The momentum correlations in the case of strong interaction undergo a smooth process of two stripes smoothly merging into a single one, i.e. the evolution of fermions into hard-core bosons.

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I. INTRODUCTION

Generally, according to quantum statistical behavior, identical particles are classified as either bosons, any number of which can occupy one single-quantum state, or fermions, which occupy a quantum state exclusively. The exchange of two fermions leads to a phase factor -1 in the total wave function due to the Pauli principle, whereas the wave function of two bosons remains the same. More than 30 years ago, a natural generalization was proposed that in two-dimensional systems there exists a third fundamental category of identical particles, anyons, which satisfy fractional statistics. According to the proposal, the overall wave function will acquire a fractional phase $e^{-i\chi\pi}$ ($0 < \chi < 1$) when two identical anyons exchange their positions. From then on, anyon has become a very important concept in condensed matter physics and has ever been successfully used in the understanding of fractional quantum Hall effect (FQHE) [1–3]. All these years, the research works on anyons remained restricted in the two-dimensional world [4, 5] until Haldane put forward the concept of fractional statistics into arbitrary dimensions [3]. Nowadays, ultracold atom systems supply as a versatile toolbox in the field of condensed matter physics and was proposed to be a wonderful candidate to realize fractional statistics. Particularly, a proposal to realize anyons in one-dimensional (1D) optical lattices has already been put forward in Ref. [6]. This opens the way to investigate the exotic properties of anyons in one-dimensional optical lattices both theoretically and experimentally.

By far, most of the theoretical works [7–10] on anyon gases in one-dimensional focus on the ground state properties, like energy, density profiles, momentum distribution, occupation distribution and occupations of the lowest natural orbital for different statistical parameters. Little attention has been paid to

anyon properties in other aspects. In this work, we will study the quantum walks (QWs) of two identical anyons in one-dimensional optical lattices. Quantum walks [11] as the quantum analogs of classical random walks is a very interesting topic deserving intensive investigations. It forms the basis of quantum efficient algorithms and provides a universal platform for quantum computations research. Compared with classical random walks, quantum walks display several intriguing non-classical features, such as superpositions and interference features, which have potential applications in universal quantum computation [12, 13] and detection of topological states [14, 15] and bound states [16]. What's more, besides single-particle quantum walks [17], multi-particle quantum walks has attracted more and more attentions. By employing non-classical correlations [18], multi-particle quantum walks bring new benefits to practical quantum technologies. Among these, the two-body quantum walks are of special interest, which have been demonstrated with both non-interacting photons in linear waveguide arrays [19–21] and interacting photons in nonlinear waveguide arrays [22]. And the coexistence of free and bound states has been explicitly observed through the quantum walks of two atomic spin-impurities in one-dimensional (1D) optical lattice (OL) [16]. Very recently, there is a theoretical work investigating two-body quantum walks [23] of bosons, fermions and hard-core bosons. Specifically, in this paper we shall study the quantum walks of two identical anyons confined in one-dimensional optical lattices with both on-site and nearest-neighbor interactions. By using generalized Jordan-Wigner transformations, we first introduce a mapping from anyons to bosons or fermions, and then calculate the two-body correlations in both position and momentum spaces. One thing need to mention here is that along the line of the excellent experimental work on Mott insulator [24], anyons can be simulated using bosons [6] with occupation-dependent hopping amplitudes, which can be realized by assisted Raman tunneling.

The paper is organized as follows. In Sec. II, we introduce the anyon lattice model with both on-

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site and nearest-neighbor interactions and construct the Hilbert space for anyon walkers. The two-body correlations in both position and momentum spaces are calculated for different evolution time, interaction strength and statistical parameter and the results are shown in Sec. III. In Sec. IV, we turn to an alternative definition of the anyon and study the correlation property of the so-called hard-core anyons. A brief summary is given in Sec. V.

II. MODEL AND METHOD

We consider quantum walks of two indistinguishable anyons in a 1D optical lattice described by the Hamiltonian with periodic boundary condition

$$H^a = \sum_{l=-L}^L \left[-J \left(a_l^\dagger a_{l+1} + h.c \right) + \frac{U}{2} n_l (n_l - 1) + V n_l n_{l+1} \right]. \quad (1)$$

Here a_l^\dagger (a_l) creates (annihilates) an anyon on the l -th site, $n_l = a_l^\dagger a_l$ is the particle number, J is the hopping between neighboring sites, U and V describe the on-site and nearest-neighbor interactions, respectively. We consider the initial condition being that two particles are localized at adjacent lattice sites and the dynamics on $2L + 1$ lattice sites represents a typical QW problem of continuous time.

The commutation relations [25–27] (CRs) for the anyonic operators read as

$$\begin{aligned} a_l a_k^\dagger &= e^{-i\chi\pi\epsilon(l-k)} a_k^\dagger a_l + \delta_{lk}, \\ a_l a_k &= e^{i\chi\pi\epsilon(l-k)} a_k a_l, \\ a_l^\dagger a_k^\dagger &= e^{i\chi\pi\epsilon(l-k)} a_k^\dagger a_l^\dagger. \end{aligned} \quad (2)$$

The sign function $\epsilon(x)$ gives $-1, 0$, or 1 depending on whether x is negative, zero, or positive. To describe the fractional statistics of the anyon, it is evident that the range of parameter χ is sufficient to be restricted in the interval $\chi \in [0, 1]$. In the original work [25] introducing anyonic system model, the anyonic fields were usually realized in terms of the bosonic fields. This assures that the anyonic system reduces to the bosonic system naturally in the limit $\chi = 0$. The same idea, called the anyon-boson mapping, will be used below. However, anyons with statistics $\chi = 1$ are pseudo-fermions: being fermions off-site, they are nevertheless bosons on-site. An alternative realization of anyonic fields in terms of the fermionic fields was proposed to eliminate the difficulties for zero-range interaction [28]. This fermionic representation with appropriate modification of the statistical parameter χ makes it possible to describe only the infinite repulsive limit, i.e. the hard-core interaction, which will be adopted to study the correlation of hard core anyons in Sec. IV.

In order to study the correlation property of anyons, we resort to an exact mapping between anyons and

bosons in 1D. Let us introduce the fractional version of a Jordan–Wigner transformation [6]

$$\begin{aligned} a_l &= b_l \exp \left(-i\chi\pi \sum_{i=-L}^{l-1} n_i \right), \\ a_l^\dagger &= \exp \left(i\chi\pi \sum_{i=-L}^{l-1} n_i \right) b_l^\dagger, \end{aligned} \quad (3)$$

with $n_l = a_l^\dagger a_l = b_l^\dagger b_l$ the number operator for both particle types. Starting from the bosonic CRs, i.e. $[b_l, b_k] = [b_l^\dagger, b_k^\dagger] = 0$, and $[b_l, b_k^\dagger] = \delta_{lk}$, we can validate that the mapped operators a_l actually obey the anyonic commutation relations as introduced in Eq. (2). This mapping elucidates that anyons in 1D are indeed non-local quasi-particles, made of bosons with an attached string operator.

Our final goal is to propose a realistic method for studying the dynamics of an interacting gas of anyons in 1D OLs. Hence, by means of the anyon–boson mapping (3), and considering the periodic boundary condition of anyons in 1D lattices [31],

$$\begin{aligned} a_L^\dagger a_{-L} &= \exp \left(i\chi\pi \sum_{i=-L}^{L-1} n_i \right) b_L^\dagger b_{-L}, \\ a_{-L}^\dagger a_L &= b_{-L}^\dagger b_L \exp \left(-i\chi\pi \sum_{i=-L}^{L-1} n_i \right), \end{aligned} \quad (4)$$

the Hamiltonian H^a can be rewritten in terms of bosonic operators,

$$\begin{aligned} H^b &= -J \sum_{l=-L}^{L-1} \left(b_l^\dagger \exp(-i\chi\pi n_l) b_{l+1} + h.c \right) \\ &\quad - J \left(\exp \left(i\chi\pi \sum_{i=-L}^{L-1} n_i \right) b_L^\dagger b_{-L} + h.c \right) \\ &\quad + \frac{U}{2} \sum_{l=-L}^L n_l (n_l - 1) + V \sum_{l=-L}^L n_l n_{l+1}. \end{aligned} \quad (5)$$

The mapped, bosonic Hamiltonian thus describes bosons with an occupation-dependent amplitude $J \exp(-i\chi\pi n_l)$ for hopping processes between adjacent sites $(l, l+1)$ except on the boundary $(-L$ and $L)$. If the target site l is unoccupied, the hopping amplitude is merely J . If it is occupied by one boson, the amplitude reads $J \exp(-i\chi\pi)$, and so on. We underline that the non-local mapping between anyons and bosons, Eq. (3), leads luckily to a purely local, and thus viable Hamiltonian. As expected from anyons, the reflection parity symmetry is broken at the level of the commutation relations (2). The fractional Jordan–Wigner transformation (3) transfers this asymmetry also to the bosonic case: the resulting Hamiltonian (5) features a phase factor acting only on the target site l and thus violates parity.

We now discuss the Hilbert space involved by the QWs of two particles. Since $[N, H] = 0$, the total particle number N is conserved and

the system will evolve in the two-particle Hilbert space. For two anyons, their Hilbert space can be spanned by basis, $B_a^{(2)} = B_b^{(2)} = \left\{ |l_1 l_2\rangle = (1 + \delta_{l_1 l_2})^{-\frac{1}{2}} b_{l_1}^\dagger b_{l_2}^\dagger |\mathbf{0}\rangle, -L \leq l_1 \leq l_2 \leq L \right\}$ because of the exact mapping between anyons and bosons. Here, $|\mathbf{0}\rangle$ denotes the vacuum state. Given $B_b^{(2)}$, it is easy to construct the Hamiltonian matrix $H^{(2)}$ in two-particle sector. In units of $\hbar = 1$, the time evolution of an arbitrary state obeys

$$i \frac{d}{dt} |\psi(t)\rangle = H^{(2)} |\psi(t)\rangle, \quad (6)$$

with $|\psi(t)\rangle = \sum_{l_1 \leq l_2} C_{l_1, l_2}(t) |l_1 l_2\rangle$ for anyons. Below we will study the continuous-time QWs of two anyons induced by the statistics parameter χ starting from an initial state $|\psi_{\text{initial}}\rangle = a_0^\dagger a_1^\dagger |\mathbf{0}\rangle$, where the two anyons are prepared in two adjacent sites 0 and 1. In order to explore the correlation between two quantum walkers, we calculate the two-body correlation in position space,

$$\Gamma_{qr}(t) = \langle \psi(t) | a_q^\dagger a_r^\dagger a_r a_q | \psi(t) \rangle, \quad (7)$$

and that in momentum space,

$$\Gamma_{\alpha\beta}(t) = \langle \psi(t) | a_\alpha^\dagger a_\beta^\dagger a_\beta a_\alpha | \psi(t) \rangle. \quad (8)$$

The operators in momentum space are defined by the discrete Fourier transformation

$$\begin{aligned} a_\alpha^\dagger &= \frac{1}{\sqrt{2L+1}} \sum_{l=-L}^L e^{-ip_\alpha l} a_l^\dagger, \\ a_\alpha &= \frac{1}{\sqrt{2L+1}} \sum_{l=-L}^L e^{ip_\alpha l} a_l, \end{aligned} \quad (9)$$

with the quasi-momentum $p_\alpha = \frac{2\pi\alpha}{2L+1}$ and the indices $q, r, \alpha, \beta = (-L, \dots, 0, \dots, L)$. The correlation matrix $\Gamma_{qr}(t)$ represents the probability of detecting one particle at site q and its twin particle at site r , which is calculated after different evolution times Jt for different values of the interaction strengths V or U . Similar probability interpretation is imposed on $\Gamma_{\alpha\beta}(t)$, however, in the momentum space. At all stages the particles are far from the lattice boundaries - the signal due to the boundary condition will be discussed elsewhere. The correlations in the following figures are rescaled by their maximum values such that $\Gamma_{qr}(t)/\Gamma_{qr}^{\text{max}}(t)$ and $\Gamma_{\alpha\beta}(t)/\Gamma_{\alpha\beta}^{\text{max}}(t)$ are shown.

III. CORRELATION OF TWO INTERACTING ANYONS

We first investigate the two-body correlations in both position and momentum spaces in absence of the on-site interaction, i.e. $U/J = 0$. To be more specific, we let the nearest-neighbor interaction be attractive $V < 0$ in the whole paper. Following the procedure described in Sec. II, the two-body correlations in position and momentum spaces can be exactly obtained

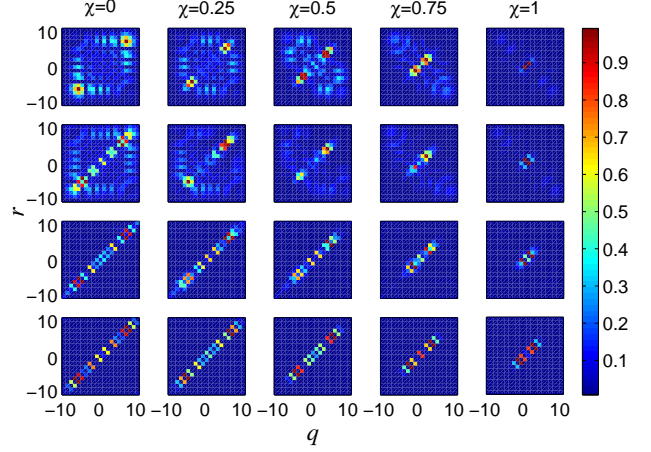


FIG. 1: (Color online) Two-body correlations of anyonic quantum walkers in position space. The nearest-neighbor interaction strength $|V/J| = 0, 1, 4$ and 80 from top to bottom and the on-site interaction strength $U/J = 0$. Here we only show the instantaneous correlations before colliding with the boundaries $L = \pm 10$. The corresponding evolution times are $Jt = 4, 4.5, 7.5$ and 110 from top to bottom. For each row, $\chi = 0, 0.25, 0.5, 0.75, 1$ (from left to right).

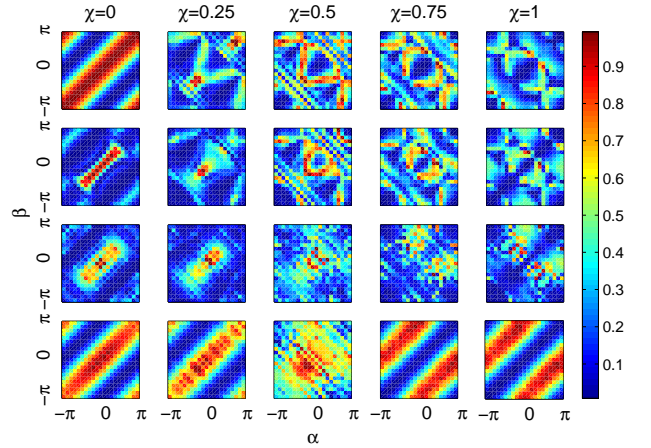


FIG. 2: (Color online) Two-body correlations of anyonic quantum walkers in momentum space with the same parameters as in Fig. 1. Note that spatial asymmetry in the bottom panel for $\chi = 1$ persists for very strong interaction.

and are shown in Fig. 1 and Fig. 2. The effects induced by the statistical parameter χ provide clear insights into the exotic behavior of anyonic two-body QWs.

As shown in Fig. 1, the first column is corresponding to the Bose limit ($\chi = 0$), where the obvious bunching behavior appears in the two-body correlations. This is exactly what has been shown in [23]. Two bunching points along the principal diagonal line get closer and closer with increasing χ . The last column is for the Fermi limit ($\chi = 1$), where the expected anti-bunching behavior is not seen. This is due to that the Fermi limit here corresponds to pseudo-fermions instead of real fermion walkers. Pseudo-fermions in

our system behave as follows: being bosons on-site, they tend to stay together in any one of the initial sites when there is no interaction; being fermions off-site, they start to occupy adjacent lattice sites for finite interaction V and stick together when co-walking in opposite directions with equal probability. For strong interaction V , correlations due to the independent walking become invisible and the real fermion behavior returns [23] (see the lower-right panel in Fig. 1).

Furthermore, we find that the correlations in the Bose limit ($\chi = 0$) and in the Fermi limit ($\chi = 1$) are both symmetric about the initial positions of two anyons. However, in the presence of a finite strength of nearest-neighbor interaction (the second and third rows in Fig. 1), the correlations are found to be asymmetric once the statistical parameter χ deviates from the two limits. This can be understood as a pure effect of the statistical parameter χ . With increasing χ the occupation-dependent statistical factor in (5) becomes more and more important: the tunneling processes connecting sites with different occupations will contribute different values. We see difference when $n_l = 1$: the hopping term to the left will gain a phase factor $e^{-i\chi\pi}$, the term to the right, on the other hand, acquires $e^{i\chi\pi}$. In the kinetic part of the Hamiltonian this incoherent superpositions is amplified by an increasing χ and induces the asymmetry of the correlation. The attractive interaction between nearest-neighboring sites forces the two anyons co-walking together indicated as the significant correlations at the two secondary diagonal lines. Stronger interaction dominates the dynamics and the bunching and co-walking behavior revives as shown in the last row in Fig. 1.

The two-body correlations in momentum space are shown in Fig. 2. The first column of Fig. 2 recovers again the behavior of bosonic walkers in Ref. [23], while the last column fails to converge to the fermi limit due to the pseudo-fermion nature of the anyons when $\chi = 1$. Compared with the position space in Fig. 1, the effects of statistical parameter χ on the asymmetry of the two-body correlations are much clearer in momentum space. In Fig. 2, we find that once the statistical parameter χ deviates from the Bose limit ($\chi = 0$), the interesting asymmetry emerges immediately in the two-body correlations - we even do not need finite interaction strength. It is thus concluded that the asymmetry originates from the exotic statistical properties of the anyons. For strong enough interaction, the spatial correlations of two anyonic walkers degenerate into the real fermion walkers successfully, while in momentum space we find clear signature of crossover from bunching behavior at $\chi = 0$ to anti-bunching behavior at $\chi = 1$ but the asymmetry persists for very strong interaction as can be seen in the last row of Fig. 2.

Next we introduce the on-site interaction U and see how it will compete with the nearest-neighbor interaction V . In Fig. 3, V is fixed to $|V/J| = 1$ and we vary U in a range from 0 to 80 in order to shed a light on the different roles played by the two interaction terms in the Hamiltonian. The first column

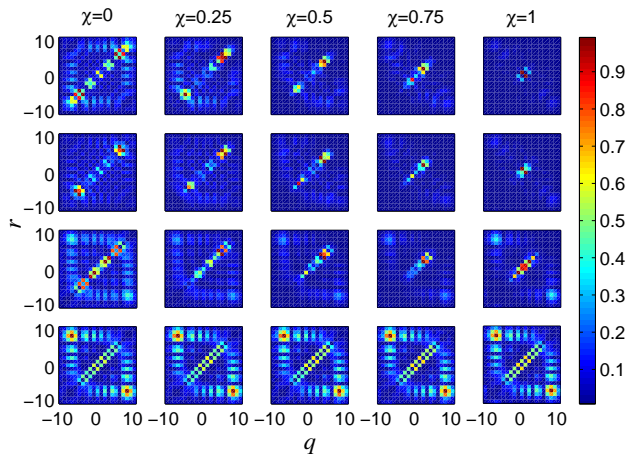


FIG. 3: (Color online) Two-body correlations of anyonic quantum walkers in position space. The on-site interactions are $U/J = 0, 1, 4$ and 80 (from top to bottom) with fixed $|V/J| = 1$ and the corresponding evolution time is given by $Jt = 4.5$.

of Fig. 3 shows the Bose limit and we find that as on-site interaction U increases, the two-body correlations evolve from a bunching behavior (bosonic walkers) to an anti-bunching behavior (hard-core bosonic walkers). Similar shrinking tendency is observed with increasing statistical parameter χ . In the Fermi limit, we see that the behavior of the two-body correlations evolves from pseudo-fermions to real fermions. With large enough U , the two-body correlations of the hard-core bosons, hard-core anyons and real fermions are almost identical in the last row of Fig. 3.

It is also interesting to show the effect of the statistics parameter χ on the correlation fluctuation [29] defined as $\Gamma_{qr}^F(t) = \langle a_q^\dagger a_r^\dagger a_r a_q \rangle - \frac{1}{2} \langle a_q^\dagger a_q \rangle \cdot \langle a_r^\dagger a_r \rangle = \Gamma_{qr} - \frac{1}{2} \langle n_q \rangle \cdot \langle n_r \rangle$ and the particle density $\langle n_q \rangle = \langle a_q^\dagger a_q \rangle$. In Fig. 4, we find that the statistical parameter χ breaks the symmetry in both the correlation fluctuation and the particle density for finite U or V . The statistics parameter plays an essential role in the symmetry of the density distribution, in contrast with the case of hard-core anyons [10] where the statistical factor makes no difference. The asymmetry here again arises from the statistics-dependent hopping term in Hamiltonian (5).

IV. CORRELATION OF TWO HARD-CORE ANYONS

In view of the above-mentioned facts, i.e. two-body correlation calculated through the mapping between anyons and bosons fails to converge to the Fermi limit due to the pseudo-fermion nature of the anyons for $\chi = 1$, we turn to an alternative realization of anyonic fields in terms of the fermionic fields [28]. The mapping now is between anyons and fermions, which guarantees that anyons would return to fermions in the limit $\kappa = 0$. Here we introduce another param-

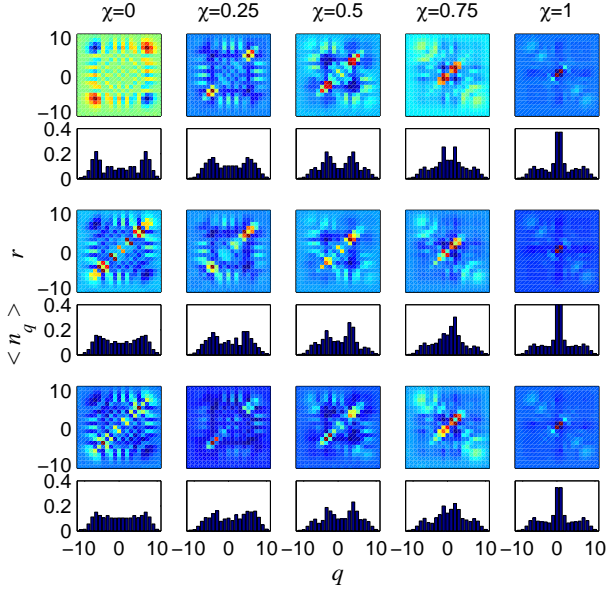


FIG. 4: (Color online) Two-body correlation fluctuations and the density distributions in position space. The on-site and nearest-neighbor interaction strengths are $(U/J, |V/J|) = (0, 0)$, $(0, 1)$ and $(1, 0)$ from top to bottom. The evolution time is given by $Jt = 4$.

ter κ to describe the statistical property of this newly defined anyon. The anyonic operators now satisfy CRs [9, 10, 28, 30]

$$\begin{aligned} a_l a_k^\dagger &= \delta_{lk} - e^{-i\kappa\pi\epsilon(l-k)} a_k^\dagger a_l, \\ a_l a_k &= -e^{i\kappa\pi\epsilon(l-k)} a_k a_l, \\ a_l^\dagger a_k^\dagger &= -e^{i\kappa\pi\epsilon(l-k)} a_k^\dagger a_l^\dagger. \end{aligned} \quad (10)$$

Then the exclusion principle $a_l^{\dagger 2} = a_l^2 = 0$ and $\{a_l, a_l^\dagger\} = 1$ follow from $\epsilon(x) = 0$. They are the commutation relations which are frequently used to describe hard-core condition [31]. Anyons with these properties are called hard-core anyons (HCAs) [30]. The CRs (10) connect real fermions and hard-core bosons when $\kappa = 0$ and 1, just as CRs (2) link ordinary bosons and pseudo-fermions when $\chi = 0$ and 1. In this section we essentially investigate the quantum walks of two HCAs in 1D optical lattice. The Hamiltonian of hard-core anyons reads

$$H^{HCA} = -J \sum_{l=-L}^L (a_l^\dagger a_{l+1} + h.c.) + V \sum_{l=-L}^L n_l n_{l+1}. \quad (11)$$

This model can also be exactly solved through a generalized Jordan-Wigner transformation mapping the hard-core anyons to spinless fermions [28],

$$\begin{aligned} a_l &= f_l \exp \left(-i\kappa\pi \sum_{i=-L}^{l-1} n_i \right), \\ a_l^\dagger &= \exp \left(i\kappa\pi \sum_{i=-L}^{l-1} n_i \right) f_l^\dagger, \end{aligned} \quad (12)$$

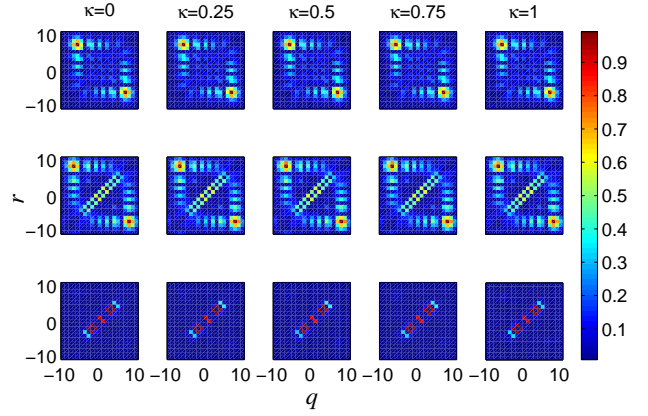


FIG. 5: (Color online) Two-body correlations in position space for HCAs. The nearest-neighbor interaction strength $|V/J| = 0, 1$, and 4 from top to bottom and the corresponding times are given by $Jt = 4, 4.5$, and 7.5, respectively. In each row, $\kappa = 0, 0.25, 0.5, 0.75, 1$ (from left to right).

where f_l^\dagger and f_l are creation and annihilation operators for spinless fermions, $n_l = a_l^\dagger a_l = f_l^\dagger f_l$ is particle number operator for both particle types. By means of this mapping, the hard-core anyonic Hamiltonian (11) can be described by fermion operators,

$$\begin{aligned} H^f &= -J \sum_{l=-L}^{L-1} (f_l^\dagger f_{l+1} + h.c.) \\ &\quad - J \left(\exp \left(i\kappa\pi \sum_{i=-L}^{L-1} n_i \right) f_L^\dagger f_{-L} + h.c. \right) \\ &\quad + V \sum_{l=-L}^L n_l n_{l+1}, \end{aligned} \quad (13)$$

on which the periodic boundary condition is imposed. We notice that a big difference here is that the statistical factor appears only on the boundary due to the hard core constraint. The corresponding Hilbert space of the two quantum walkers can be constructed as $B_{HCA}^{(2)} = B_f^{(2)} = \{|l_1 l_2\rangle = f_{l_1}^\dagger f_{l_2}^\dagger |0\rangle, -L \leq l_1 < l_2 \leq L\}$. Again the initial state is chosen as $|\psi_{initial}\rangle = a_0^\dagger a_1^\dagger |0\rangle$.

By the similar procedure described in Sec. II, we calculate the two-body correlations both in position and momentum spaces for HCAs. In position space, as shown in Fig. 5, we find that the two-body correlations echo no signal of the statistical parameter κ and are all the same as those for spinless fermions [23]. In each row, from left to right, the pictures are almost the same to naked eye though slight change happens according to the numerical data. This is because the Hamiltonian Eq. (13) is the same as spinless fermions except the boundary terms. With increasing V the anyons show uniform transition from anti-bunching to co-walking regardless of the statistical factor. The

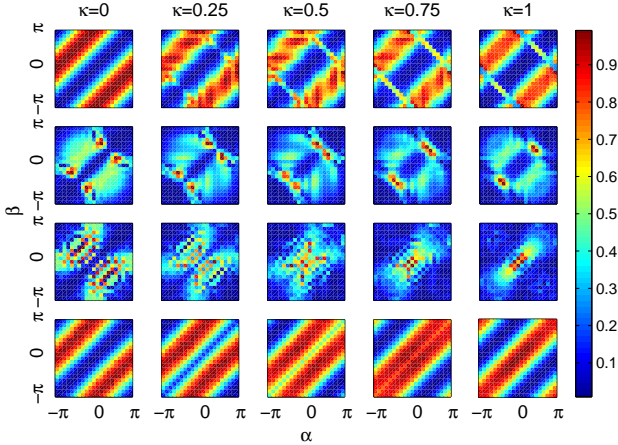


FIG. 6: (Color online) Two-body correlations in momentum space for HCAs. The nearest-neighbor interactions are $|V/J| = 0, 1, 4$ and 80 from top to bottom. And the corresponding evolution times are given by $Jt = 4, 4.5, 7.5$, and 110 , respectively. The figures in the first and last column are exactly the same as those in [23].

diagonal elements of the correlation vanish due to the hard core condition, which can be seen from

$$\begin{aligned} \Gamma_{qr} &= \langle \psi^{HCA}(t) | a_q^\dagger a_r^\dagger a_r a_q | \psi^{HCA}(t) \rangle \\ &= \langle \psi^{HCA}(t) | n_q n_r | \psi^{HCA}(t) \rangle, \end{aligned} \quad (14)$$

where the statistical factors cancel with each other automatically. However, this is not the case for two-body correlations in momentum space. To obtain the correlation $\Gamma_{\alpha\beta}(t) = \langle \psi^{HCA}(t) | a_\alpha^\dagger a_\beta^\dagger a_\beta a_\alpha | \psi^{HCA}(t) \rangle$, one has to calculate terms like $\langle \psi^{HCA}(t) | a_p^\dagger a_q^\dagger a_r a_s | \psi^{HCA}(t) \rangle$, where the four indices are generally different. Thereby, the statistical factors introduced by the mapping Eq. (12) will stay. The two-body correlations in momentum space carry more informations as shown in Fig. 6.

In Fig. 6, the first and last columns correspond precisely to the Fermi limit $\kappa = 0$ for real fermions and the Bose limit $\kappa = 1$ for hard-core bosons, respectively, which are the main topic of Ref. [23]. Once the statistical factor κ deviates from the two limits, the asymmetry of correlation reappears and in between there again exists a crossover from an anti-bunching behavior (fermionic walkers) to a bunching behavior (hard-core bosonic walkers) with increasing κ . Especially, for strong interaction, the last row shows how the two stripes in the momentum correlation smoothly merge into a single one. This provides a way to probe

different statistics from the observations of the correlations in momentum space, which is viable in ultracold atom laboratories nowadays.

V. CONCLUSIONS

In summary, we have investigated the two-body correlations of 1D quantum gas of anyons confined in optical lattices with both on-site and nearest-neighbor interactions using exact numerical method. With Jordan-Wigner transformation the anyon lattice model is mapped to bosonic one and thus the Hilbert space of anyons can be constructed from that of bosons. Then by solving the time-dependent Schrödinger equation we obtain the wave function in arbitrary evolution time. Numerical results show that the anyonic two-body correlations in position space exhibit distinct properties from the bosons and fermions. In the Bose and Fermi limits the correlations are symmetric about the initial positions of walkers in position space. The variation in statistic parameter χ drives the system from Bose statistics to Fermi statistics and the fractional statistics in between. An interesting asymmetry arises in the correlations in both position and momentum spaces due to the fractional statistics of the anyons. The anyon-boson mapping links the bosons and pseudo-fermions when $\chi = 0$ and 1 , respectively, while for HCAs, the anyon-fermion mapping connects the fermions and hard-core bosons for $\kappa = 0$ and 1 . In this sense we conclude that anyons realized in the two ways are not simply intermediate particles between bosons and fermions. In either case, the correlations only converge to one limit, through which the anyons are defined. Stronger inter-site interaction is needed in achieving perfect evolution from bosonic walkers to fermionic walkers with increasing χ . On the other hand, stronger on-site interaction will put a hard-core constraint on anyons, the statistics of which can be distinguished from the correlation in momentum space.

Acknowledgments

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